

Measurement

3.1 Introduction

The importance of measurement

No matter what the context, the basic idea of measurement is to enable *quantitative comparison* of one property or thing with another. The ability to numerically compare quantities may be considered one of the defining characteristics of a science: without measurement, there is only speculation.

This book contains a chapter on measurement because in the teaching of physics, the insights achieved as a result of studying measurements are often taught as secondary to insights derived from theoretical approaches to a subject. Indeed, I have attended entire lecture courses where no reference was made to a single measurement in the outside world. However, in practice it is exceedingly rare for physicists to proceed in such a fashion. The beliefs that physicists hold about the world are, in a sense, *forced upon them* by measurements. The results of measurements act as guides, and constraints, for developing explanatory ideas about the world. So familiarity with measurements and the way they are made is as important an aspect of physics as familiarity with (say) mathematics. Although a student might be forgiven for thinking otherwise, it is not mathematics but measurement, that makes physics a science rather than a philosophy.

This chapter is divided into four sections:

- §3.2 **Units:** Measurement units are in some ways a most mundane subject. Units are the standard quantities which enable the quantitative comparison, of which I spoke above, to be realised. However in other ways things are not so simple.
- §3.3 **Key measurement techniques:** Given the importance of measurement, the techniques used to carry out this process are of considerable interest. Here we look at some of the more common techniques. Throughout the twentieth century there has been astonishing progress in techniques driven by the availability of specific technological tools. Three trends stand out above all others: the exploitation of the phenomenal accuracy with which *time* may be measured; the use of digital voltmeters and the concept of a sensor; and most recently the use of lasers and opto-electronic techniques.
- §3.4 **Environments:** In considering measurements of the properties of matter we are interested in how matter behaves in differing environments, for example high and low temperatures. In this section we see in rough terms how these environments are created.
- §3.5 **Uncertainty:** All measurements have an associated uncertainty. In this section we mention the uncertainty associated with the extensive tabulations and graphs in this book.

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You will find copies of the figures and tables used in this chapter, along with links to several of the world's leading National Measurement Institutes mentioned in the chapter.

3.2 Units

Recalling the fundamental idea of a measurement as a comparison, *units* act as *standard comparison quantities*. This notion of a unit is apparent in the names given to historical units of length such as the *foot*. These units referred to the length of a standard object against which all else in the ‘kingdom’ would be compared. Unfortunately, neighbouring kingdoms frequently had different definitions for quantities which made *quantitative comparison* exceedingly difficult. For example, in 1686, Newton was attempting to show *quantitatively* that his theory of gravity could explain both phenomena on the Earth as well as the motion of the Moon and other astronomical bodies. In searching for observations in support of his theory (Book III of *Principia Mathematica*) he describes the diameter of the Earth and the motion of the Moon in terms of *Paris* feet, as distinct from *English* feet used elsewhere in his work. Clearly the units of measurement are getting in the way of comparison rather than making it easier.

Historically, there have been two trends that have taken the science of measurement away from the use of units like the foot. These trends are:

- towards the use of a system of units that are agreed internationally
- towards defining standard quantities in terms of *phenomena* that can be *realised* by anyone. This is a move away from the use of defining objects or *artefacts*.

3.2.1 National standards and international agreements

Each country employs a system of *legal metrology*, the main aim of which is to facilitate trade rather than science, but which serves for both purposes. The UK has currently nearly completed adoption of the SI system of measurement units detailed below. The aim of having a coherent system of measurement units is a major scientific and management challenge.

The scientific part of the challenge is the tackled by National Measurement Institutes (NMIs): in the

UK, the National Physical Laboratory (NPL); in the USA, the National Institute of Science and Technology (NIST); and in Germany the Physikalisch Technische Bundesanstalt (PTB). These laboratories manufacture and maintain apparatus that allow the *realisation* of accurate representations of the units (Tables 3.1 to 3.3). For example, each of the above NMIs has an apparatus which produces a known voltage against which a voltmeter belonging to a customer may be calibrated. You might now ask the question ‘How do the NMIs know the value of the “known voltage” that they produce?’ The answer is that they base the value on fundamental physical laws and principles. For the case of voltage, the known voltages are produced by so-called Josephson devices. They also work with each other and NMIs worldwide to ensure that their *realisations* of measurement units are internationally consistent.

The management part of the challenge is tackled by a quality assurance organisation: in the UK, the United Kingdom Accreditation Service (UKAS). This organisation ensures that laboratories bearing its seal of approval carry out valid calibrations and do not claim measurement uncertainties lower than they can actually achieve. Part of this validation is to ensure that relevant measuring instruments used in these laboratories are regularly recalibrated at the NPL. This system maintains a hierarchy of measurement accuracy and ensures that the lower orders of the hierarchy maintain as much as is practicable of the accuracy achieved at the apex of the hierarchy, NPL. However, at each tier of the hierarchy, extra measurement uncertainties are inevitably introduced. This makes it particularly important that the realisations of the base quantities at the NMIs have the smallest measurement uncertainties possible.

In 1999, the trend towards the international harmonisation of measurement units was cemented by the signing of the *Mutual Recognition Arrangement*. The 19 signatories agreed to recognise each other’s realisations of measurement units through a programme of key inter-comparisons.

3.2.2 Artefacts and realisations

There are two ways in which we can agree on a measurement standard. Countries can agree on either:

- a standard *object* of which there is only one, and against which all others are compared. This object is referred to in measurement circles as an *artefact*
- or
- a standard *physical phenomenon* or situation which it is practical for all countries to create. This situation is referred to in measurement circles as a *realisation* of a measurement unit.

The reason for the trend away from defining units by artefacts to definitions in terms of realisations may be seen by considering the one measurement unit that is still defined by an artefact: the kilogram.

The kilogram

There is only one kilogram. It is a unique piece of platinum–iridium alloy kept at the International Bureau of Weights and Measures (BIPM), Paris. It has its own special symbol, a gothic letter ‘K’ \mathfrak{K} , which refers the object itself and not its mass. There are 41 copies of the kilogram in existence: the United Kingdom holds copy number 18. After it was last cleaned, the UK copy of the kilogram weighed $59 \pm 3 \mu\text{g}$ more than the prototype kilogram, and then gained mass at a rate of $1 \mu\text{g}$ per month for a year and now gains mass at a rate of approximately $1 \mu\text{g}$ per year. However, the UK kilogram is so valuable that it spends its days in carefully controlled conditions inside three bell jars. Every time it is removed from its protective surrounding its mass changes a little as it reacts slowly with the air and adsorbs (or desorbs) a little moisture. Periodically the UK’s copy of the kilogram is returned to Paris to make sure that its mass has not changed too much. But what if the international prototype kilogram, \mathfrak{K} itself, changed its mass? In fact recent inter-comparisons have shown that \mathfrak{K} definitely *has* changed mass, and a small coterie of scientists keep track of its changes by inter-comparing the copies of it.

Thus defining a standard quantity in terms of an object makes life very difficult for everyone concerned, and allows for the possibility of long term

drift in the magnitude of measurement units. Definitions of standard quantities in terms of *artefacts* of any kind have died out leaving ‘standard metres’ as museum pieces. The reason for the anomalous longevity of the kilogram arises for the simple reason that despite the unsatisfactory nature of the arrangement, the resulting measurement uncertainties are lower than can be achieved by any competitive technique.

Other units are defined in terms of physical phenomena which may, in principle at least, be *realised* by anyone. The phenomena chosen are believed to be well understood, so that the definitions are unlikely to require revision, and hence the system of units will not change with time.

For example, the phenomena of the coexistence in equilibrium of water vapour, liquid water and ice defines the *triple point* of water (see Chapter 10 for more details). The temperature at which the triple point of pure water occurs is *defined* to be 273.16 kelvin. All one needs to do in order to realise this temperature is to obtain some pure water, manufacture a simple vessel, and stick it into ice at close to 0°C . After equilibration, the temperature inside is *defined* to be 273.16 kelvin. This is a physical phenomenon which may be realised with relative ease and which will not change from one year to the next, or from one place to another.

3.2.3 The international system of units

All rational physicists use the system of units known as the *International System of Units*, or *Le Système International d’Unités*, or more normally *SI units*. At least they use SI units when discussing subjects other than their own. For discussing their own field of expertise, they frequently use sets of *colloquial units*, which for historical reasons, and reasons of genuine convenience, have not died out. In this book, I have chosen always to use the SI units, adding colloquial units where appropriate.

The SI units consist of seven base or fundamental quantities in terms of which all other quantities are derived. The SI base units and definitions are given Table 3.1.

Table 3.1 The SI base units. Notice that, with the exception of the kilogram, the definitions are in terms of physical phenomena and not defining artefacts. Although the definitions seem obscure, the language is carefully chosen in order to make accurate realisations of the standards feasible.

Quantity:	
Unit (abbreviation)	Definition
Time: second (s)	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium-133 atom.
Length: metre (m)	The metre is the length of the path travelled by light in vacuum during a time interval $1/299,792,458$ of a second. Note: This defines $299,792,458 \text{ ms}^{-1}$ as the <i>exact</i> speed of light in a vacuum.
Mass: kilogram (kg)	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram
Electric Current: ampere (A)	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton, per metre of length.
Thermodynamic temperature: kelvin (K)	The kelvin, unit of thermodynamic temperature, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of pure water.
Amount of substance: mole (mol)	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.
Luminous Intensity: candela (cd)	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation 540×10^{12} hertz and that has a radiant intensity of $1/683$ watt per steradian

There are two other units (Table 3.2), called supplementary units, which are used to distinguish between quantities that are of a different nature, but which would otherwise have the same SI units. For example *angular velocity* is specified in units of rad s^{-1} even though the radian is dimensionless. This distinguishes the units of *angular velocity* from those of *frequency*.

Other units are derived from these base units, and have names given in terms of the base units involved. For example *mass density* is expressed in a unit called the *kilogram per cubic metre* and has the symbol kg m^{-3} . Other units derived from these base units are specially named in honour of various scientists. These are listed in Table 3.3.

I think it is important to realise that although SI is the culmination of a century of work in devising a rational system of units, alternative systems are still in use.

3.2.4 An example of the realisation of units: the mole

As an example of the way that SI defines the magnitude of a unit, let us consider what SI has to say about *the mole*: the unit of the *amount of substance*. First recall the definition from Table 3.1:

The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.

A supplementary note to the agreement of this definition mentions that it is important to specify which elementary entities are being referred to.

Table 3.2 SI supplementary units.

Quantity	Name		Expression in terms of SI base units
	Name	Symbol	
plane angle	radian	rad	$\text{m m}^{-1} = 1$
solid angle	steradian	sr	$\text{m}^2 \text{m}^{-2} = 1$

Table 3.3 SI derived units with special names. The name of the units are all written with lower case letters (with the exception of degree Celsius), but that the symbols for the units have upper case letters: be careful to distinguish between seimens (S) and seconds (s). The symbol for the ohm, Ω , is the greek letter 'W', called omega.

Quantity	Name	Symbol	Expression in terms of other units	Expression in terms of SI base units
frequency	hertz	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure	pascal	Pa	$N\ m^{-2}$	$m^{-1}\ kg\ s^{-2}$
stress				
energy	joule	J	$N\ m$	$m^2\ kg\ s^{-2}$
work				
quantity of heat				
power	watt	W	$J\ s^{-1}$	$m^2\ kg\ s^{-3}$
radiant flux				
electric charge	coulomb	C		$s\ A$
quantity of electricity				
electrical potential	volt	V	$W\ A^{-1}$	$m^2\ kg\ s^{-3}\ A^{-1}$
potential difference				
electromotive force				
capacitance	farad	F	$C\ V^{-1}$	$m^2\ kg^{-1}\ s^4\ A^{-1}$
electric resistance	ohm	Ω	$V\ A^{-1}$	$m^2\ kg\ s^{-3}\ A^2$
electric conductance	siemens	S	$A\ V^{-1}$	$m^2\ kg^{-1}\ s^3\ A^{-1}$
magnetic flux	weber	Wb	$V\ s$	$m^2\ kg\ s^{-2}\ A^{-1}$
magnetic flux density	tesla	T	$Wb\ m^{-2}$	$kg\ s^{-2}\ A^{-1}$
inductance	henry	H	$Wb\ A^{-1}$	$m^2\ kg\ s^{-2}\ A^{-2}$
Celsius temperature	degree celsius	$^{\circ}C$		K
luminous flux	lumen	lm		cd sr
illuminance	lux	lx	$lm\ m^{-2}$	$m^{-2}\ cd\ sr$

They may be ions, electrons, atoms, molecules, other particles or groups of particles.

Notice that this definition leaves it up to the scientific community to invent practical realisations of the definition. Let us look at a couple of ways in which, starting from first principles, we might determine the *amount of substance* in a sample of material.

Realisation #1

If the substance is a gas then we can use the fact that, as we show in Table 5.3, the volume of 1 mole of any gaseous substance held at standard temperature and pressure (STP), is close to $22.413 \times 10^{-3} m^3$. In §5.2.2 we see that we can understand *why* this is so based on an analysis of a microscopic theory of the behaviour of a gas. This analysis produces the perfect gas equation $PV = zRT$ (Eq. 4.1), while more sophisticated

analyses (§4.4.2) arrive at slightly different equations that enable us to understand the small deviations of the volume at STP from the ideal value. We thus have a piece of physics which contains no 'fudge factors'. Thus, we can use the perfect gas equation to realise any of the quantities in the perfect gas equation (P , V , z , T) in terms of other quantities.

Thus by measuring the pressure of the gas at $0^{\circ}C$ in a container of known volume V we can infer the number of moles z of gaseous substance in the container. In fact, $PV = zRT$ is more commonly used to realise the definition of temperature T in terms of the pressure P of z moles of gaseous material in a container of fixed volume V .

Realisation #2

If the material is a solid or liquid and its chemical composition is known, then the number of moles

can be determined by weighing. If we consider a material A of chemical composition X_pY_q then we note that the molar mass of A is given by:

$$M(A) = \frac{m[A]}{m[^{12}\text{C}]} \times 0.012 \text{ kg mol}^{-1} \quad (3.1)$$

Here $m(A)$ indicates the mass of a single entity of the substance A . In this case 'a single entity' refers to a collection of p atoms of type X and q atoms of type Y . Similarly $m(^{12}\text{C})$ indicates the mass of a single atom of ^{12}C . Note that although neither of these masses is known with great accuracy, the *ratio* expressed in Equation 3.1 can be measured with high accuracy by, for example, a *mass spectrometer*. Similarly the ratios:

$$\frac{m[X]}{m[^{12}\text{C}]} \text{ and } \frac{m[Y]}{m[^{12}\text{C}]} \quad (3.2)$$

can also be determined accurately. We can thus state that the mass of 1 mole of A is:

$$M(A) = \left[p \left(\frac{m[X]}{m[^{12}\text{C}]} \right) + q \left(\frac{m[Y]}{m[^{12}\text{C}]} \right) \right] \times 0.012 \text{ kg mol}^{-1} \quad (3.3)$$

This enables the quantity of elementary entities in a sample of substance A to be determined as shown in Example 3.1

Example 3.1

How many moles of calcium fluoride (CaF_2) are there in 4.3209 kg of the substance ?

For CaF_2 we write $X = \text{Ca}$, $Y = \text{F}$, $p = 1$ and $q = 2$. The ratios of Equation 3.3 are noted in *Kaye and Laby* to be:

$$\frac{m(\text{Ca})}{m(^{12}\text{C})} = \frac{40.08}{12} = 3.34$$

and

$$\frac{m(\text{F})}{m(^{12}\text{C})} = \frac{18.9984}{12} = 1.5832$$

So the mass of a mole specified by Equation 3.3 is:

$$\begin{aligned} M(\text{CaF}_2) &= [1 \times 3.34 + 2 \times 1.5832] \times 0.012 \\ &= 0.07808 \text{ kg mol}^{-1} \end{aligned}$$

If we have 4.3209 kg of CaF_2 we can determine the amount of CaF_2 by dividing the sample mass by the molar mass:

$$\begin{aligned} \text{Amount of CaF}_2 &= \frac{4.3209}{0.07808} \text{ mol} \\ &= 55.34 \text{ mol.} \end{aligned}$$

Notice that the definition of a mole does not specify which realisation should be used: this is a matter for scientists and engineers to choose. As improvements in our understanding develop, or new techniques become available, the definition need not change. But the realisations of the definition may change, and hopefully become more accurate or easy to use.

3.3 Key measurement techniques

Certain techniques of measurement occur commonly in a wide variety of situations. The commonness, and hence the importance, of these techniques make them worth mentioning separately.

3.3.1 Time

Time, or its inverse, *frequency*, can be measured with stupendous accuracy. The UK National Physical Laboratory can realise a second with an accuracy of 1 part in 10^{13} . In part this is the result of hard work, but it is also due to the fact that time

lends itself to being measured accurately. For example, through what is known as the *piezo-electric effect*, a quartz (SiO_2) crystal mechanically oscillates at a frequency dependent on its size, shape, temperature, and applied electric field. If all these quantities are stabilised, it is possible for such a crystal (costing perhaps 10 pence) to oscillate at a frequency of about 10^8 Hz stable to a few parts in 10^9 over a few hours. These oscillations can be detected and converted into a stream of electrical pulses. Because we have the technology to count

pulses extremely reliably, we can determine how many pulses occur in a given interval of time. Indeed, in measurement circles *counting* is not generally considered a measurement because it has no uncertainty associated with it! The uncertainty in our count will then be two pulses at most, one at the beginning and one at the end of the timing interval. So, by counting these pulses, we can determine elapsed times of the order of one second with a resolution of 2 parts in 10^8 . All that remains is to determine the actual frequency of the crystal by comparison with an instrument calibrated at a National Measurement Institute.

Voltage to frequency conversion

The ease with which high resolution is achieved for time and frequency measurements has caused people to seek ways to convert the quantity they wish to measure into a time measurement. The most important example of this is the development of techniques for measuring voltage, i.e. electrical potential difference. A simplified example of the conversion of a voltage measurement to a time measurement is the *integrating analogue to digital converter* (ADC). An integrating ADC (Figure 3.1) consists of three parts: an *integrator*, a *comparator* and a source of clock pulses as described above. There are three stages to the measurement:

- First, a circuit generates a current accurately proportional to an input voltage. This is arranged to charge a capacitor for a fixed period

of time, i.e. for a fixed number of clock pulses.

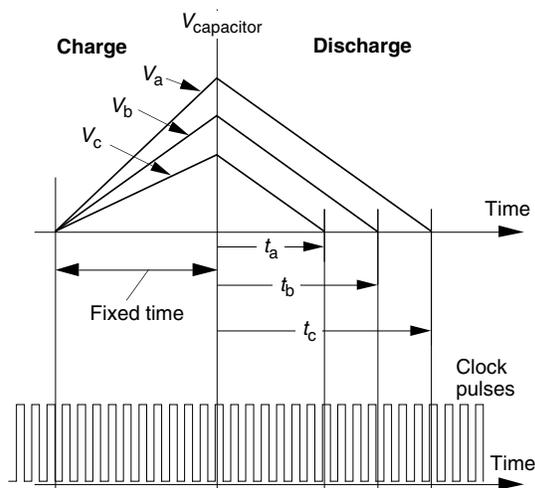
- Second, with the input disconnected, the capacitor is discharged by a constant current circuit until the voltage reaches zero, as determined by a comparator circuit. The time taken for this discharge is measured by counting the number of clock pulses that occur during the discharge. The count of clock pulses required for the capacitor to discharge has been designed to be accurately proportional to the magnitude of the input voltage. So the larger the input voltage, the longer the discharge time and hence the more pulses that are counted.
- Third, digital circuitry converts this count to the correct units. Thus a measurement of time performed by counting clock pulses has been converted into a measurement of voltage.

Many variations of this technique exist under the general heading of *charge balancing* or *voltage to frequency* conversion techniques. Example 3.2 shows the high precision which may be achieved using such techniques. In all cases the result of the measurement is now held in the state of digital circuitry and may be easily 'read' by a computer for subsequent analysis.

3.3.2 Voltage measurement and transducer technology

The accuracy and ease with which voltage can be measured has caused a trend towards the creation

Figure 3.1 An integrating analogue to digital converter.



Example 3.2

If the clock of an integrating ADC runs at 10 MHz, how accurately could a voltage be measured if the integrating period was 1 second?

If the counting is accurate to ± 1 pulse then the discharge period can be timed with an uncertainty of roughly $0.1 \mu\text{s}$. If the discharge period is roughly 0.1 s then the fractional uncertainty due to the integration is only of the order $0.1 \times 10^{-6} / 0.1 \approx 1$ part in 10^6 . If the voltage was of the order of 0.1 V this would correspond to an uncertainty of only a fraction of a microvolt. In practice, other factors make it difficult to achieve uncertainties at this low level.

of devices, known as *transducers* or *sensors*, which convert changes in physical quantities into changes in a voltage. Once a voltage that is related to the physical quantity to be measured has been generated, electrical circuits can act on the voltage to actuate a display or an alarm. Alternatively, once a voltage which corresponds to the quantity to be measured has been created, analogue-to-digital converters (Figure 3.1) can convert the voltage, and hence the quantity being measured, into a *digital code*. The data can then be conveniently analysed and stored using ever more powerful computers. Below we will look at just a few of the more important sensors.

Temperature sensors

In general, temperature sensors operate by passing an electric current through a piece of material and measuring changes in the flow of electric current as the temperature changes.

Platinum resistance thermometer

A *platinum resistance thermometer* (PRT) consists of a length of thin platinum wire. The electrical resistivity of platinum has been investigated extensively over many years and is now well documented (Figures 3.2 and 7.35). Ideally the wire should be strain-free and held delicately, but more commonly the requirement that the PRT be robust and easy to use is valued more highly than the absolute accuracy of the results. Practical PRTs are commonly formed by using a *thin film* of platinum on a substrate which conducts heat well, but is electrically insulating. Alternatively, they may be wound on a frame and then held in place by a thermally-conducting, but electrically-insulating, cement.

Platinum is chosen in preference to other metals with similar electrical properties because of its exceptional resistance to chemical corrosion. This means that the diameter of wires made of platinum does not decrease as the wire slowly corrodes. The insensitivity to corrosion allows PRTs to be used up to temperatures as high as 1000 °C.

The length and diameter of the platinum wire are commonly chosen to give the PRT a resistance of 100 Ω at 0 °C.

Thermistor

Thermistors are pieces of semiconducting materials operated in a similar fashion to PRTs, but with a dramatically different dependence on temperature (Figure 3.2). Thermistors are particularly useful because their sensitivity to temperature changes is much higher than a PRT, but only over a limited range of temperatures.

Semiconducting diodes

Semiconducting diodes are used as temperature sensors in almost exactly the same way as resistance thermometers. A constant current of typically 10 μA is passed through the diode, and the voltage across the diode is measured (Figure 3.3) Diodes are considered separately from thermistors and PRTs because at constant temperature the voltage–current characteristic is highly non-linear, or non-ohmic i.e. V is not proportional to I However, at constant current, the voltage across the diode varies linearly over a wide temperature range.

Thermocouples

A thermocouple is one of the oldest, cheapest, and most difficult to explain methods of measuring temperature. A thermocouple is an arrangement of

Figure 3.2 Schematic variation of the electrical resistance of a platinum resistance thermometer (PRT) and a thermistor. Figure 7.36 shows a more accurate representation of the electrical resistance of platinum. Notice that the resistance of a PRT increases gradually with increasing temperature. In contrast, the resistance of a thermistor falls dramatically as the temperature is increased.

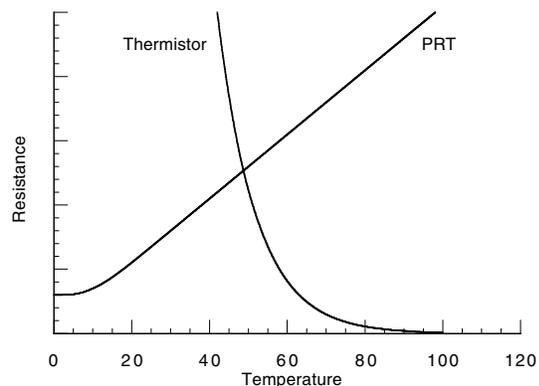
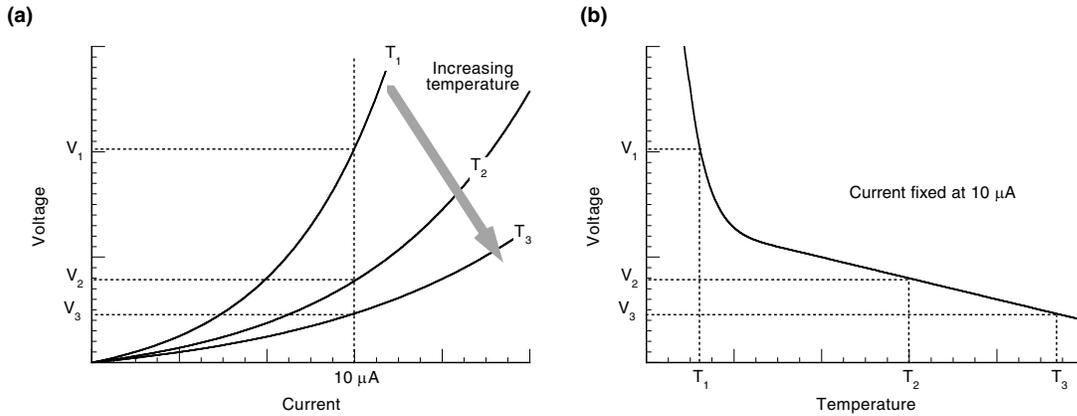
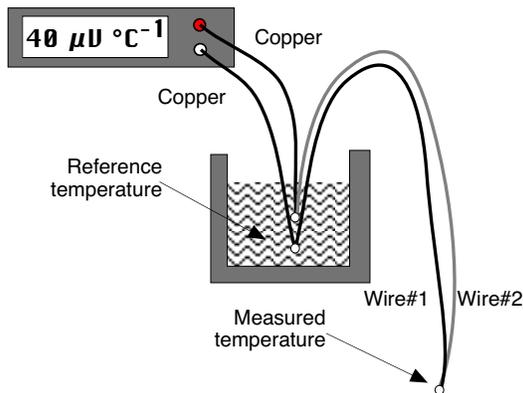


Figure 3.3. Illustration of the operation of a semiconductor diode thermometer. The variation of the voltage across the diode (a) is a non-linear, i.e. a non-ohmic function of the current through the diode. However if the current through the diode is fixed, the variation of the voltage with absolute temperature (b) is surprisingly linear over a wide temperature range.



two metals, generally in wire form, and commonly configured as shown in Figure 3.4. In the arrangement shown, a potential difference (voltage) is generated across the terminals of the thermocouple, which is related to the temperature *difference* between the two junctions. For commonly

Figure 3.4 Illustration of a common configuration for the use of a thermocouple. The thermocouple junction is placed where the temperature needs to be measured, and the other ends of the wires are joined to copper wires in a reference bath held at a known, stable temperature (typically that of melting ice). The thermocouple voltage (typically $40 \mu\text{V}$ for each degree celsius difference between the junction temperature and the reference temperature) is measured by a high-resolution voltmeter or analogue to digital converter.



chosen materials, this voltage is nearly linearly proportional to the temperature difference. If one junction is held at a reference temperature (commonly 0°C), then the temperature in Celsius of the second junction can be determined.

In practice thermocouples represent a cheap and relatively easy way to determine temperature. The metals in a thermocouple can be chosen to enable temperature measurement from a few kelvin to over 2000°C (but not with the same thermocouple). However, because of difficulties with calibration, thermocouples are rarely used where the minimum uncertainty in temperature is required. Their main advantages are their low cost and the low mass of the sensing junction

Physically, it is a curious fact that the measured thermovoltage is *not* generated at the measurement junction. Instead, the measured voltage results from any region of the *entire length* of the wires in the circuit in which there is a *temperature gradient*. The magnitude of the thermovoltage generated in each length of wire depends on (a) the composition of the wire and (b) the integral of the temperature gradients along the wire. It is really rather surprising that such complex physics results in a simple and easy to use device. The theory of thermocouples is outlined further at the end of this chapter (§3.6 Exercise P6).

3.3.3 Lasers and optical techniques

What is special about lasers?

The word *laser* has now joined the mainstream of the English language. Until relatively recently however, it was merely an acronym, LASER, standing for *Light Amplification by the Stimulated Emission of Radiation*. The basic idea of laser operation is summarised in Figure 3.5. The discovery of the phenomenon of laser action in 1960 marked the start of a revolution. Since then, techniques have been developed to exploit the interaction of laser light with matter to determine all kinds of properties of matter. Lasers have many properties as light sources that are unmatched by any other source. For example:

- Lasers are very bright: this makes the beams easy to detect
- Laser light beams have (or can have) a very low divergence. This makes it possible to control the region of interaction between a laser beam and a substance being probed
- Laser light is (or can be) monochromatic
- Laser light is (or can be) coherent
- Lasers can produce very short pulses of light
- Lasers are very intense

- Laser light in the infra-red region of the spectrum can be transmitted through optical fibres with extremely low losses.

In different applications, different properties of lasers are exploited. However, the combination of properties available makes laser techniques extremely powerful. Importantly, such techniques can permit measurements in which there is no physical contact with the substance being investigated.

Light intensity transducers

The importance of optical techniques, not least for fibre-optic communication systems, has led to many developments in the field of transducers that convert variations in light intensity into electrical signals.

Photoresistors

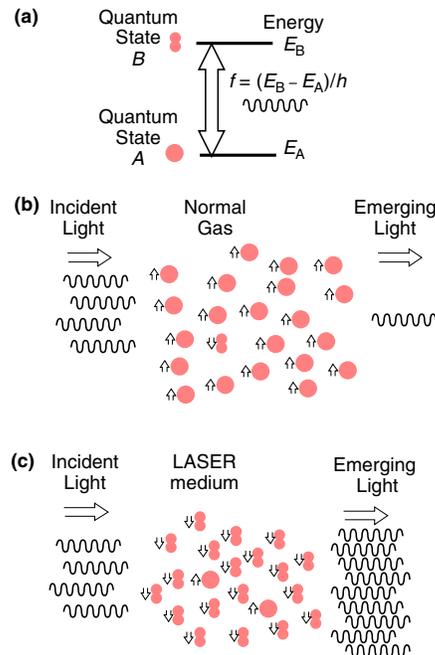
The earliest of these detectors consisted of a resistor made from the element selenium. When illuminated the electrical resistance of the selenium falls dramatically. So if a constant current is passed through a selenium resistor, the voltage across the resistor will fall at high light levels and

Figure 3.5 The principle of laser action. (a) Consider a gas of atoms each of which has two quantum states which we can refer to as A and B.

(b) The relative proportions of atoms in the quantum states depends on their energies (E_A , E_B) and is given by a Boltzmann factor $\exp[-(E_B - E_A)/k_B T]$. This ensures that *in equilibrium* there are always more atoms in quantum state A (with lower energy) than quantum state B (with higher energy). When light of just the correct frequency $f = (E_B - E_A)/h$ is shone through the gas, transitions between quantum states are stimulated *with equal probability in either direction*. However, because there are many more atoms in quantum state A with lower energy, there are many more upward transitions ($A \Rightarrow B$) than downward transitions ($B \Rightarrow A$) so on average light is absorbed by the gas.

(c) In a laser, a non-equilibrium situation is arranged by one of many techniques in which there is a *population inversion* where many more atoms are in quantum state B with higher energy. Now the passage of light of just the correct frequency stimulates more *downward* than *upward* transitions and *on average* the medium emits more light at frequency f than initially entered the gas: Hence the name *Light Amplification by the Stimulated Emission of Radiation*

To create a population inversion it is in fact necessary that there be at least one other quantum state with energy greater than E_B , but I have not shown that state on these figures.



rise at low light levels. The microscopic processes that cause the change in resistance are discussed in Chapter 7, along with the optical behaviour of the better-known semiconducting elements silicon and germanium. The photo-resistive response is generally rather slow, and except in the cheapest applications, photoresistors have now been superseded by silicon photodiodes. The exception to this is the detection of far infrared light, with wavelengths greater than around 2 microns where photoresistors have a superior performance.

Photodiodes

Currently, *photodiodes* provide the most common elements in light detecting electrical circuits. The operation of the devices is complex, but may be summarised as follows. A *photodiode* is manufactured out of a semiconductor in which there is a junction between regions with different impurities. The impurities are chosen so that at the interface between the different regions, there arises a thin region of intense electric field known as the *depletion layer*. In a photodiode, this region is arranged to occupy as large a volume as possible.

When light interacts with valence electrons in a semiconductor, the electrons may become detached from a particular atom and made available to travel through the semiconductor. Normally, the electron is drawn back to the atom it has come from by the residual positive electric charge that it leaves behind it. However if the valence electron that absorbs the energy is located in a strong electric field, such as that which exists within the *depletion layer*, then it can be drawn away from its parent atom. It then contributes to an electric current that has been induced by the interaction of light with the semiconductor.

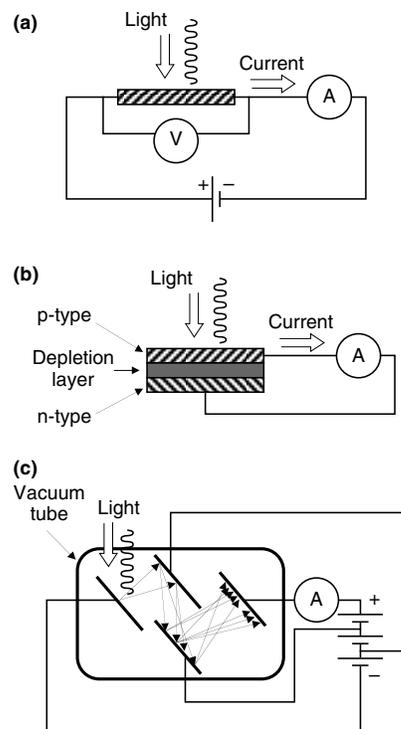
If photodiodes could operate perfectly they would generate one electron for each *photon* that was absorbed. By measuring the current generated between the two terminals of the diode we can determine the number of photons arriving at the *depletion layer* between the two impurity regions of the semiconductor. Importantly, photodiodes are most commonly made of silicon, which can be interfaced directly with silicon integrated circuits.

Figure 3.6 Light Transducers. Devices that generate a voltage or current when illuminated are technologically and experimentally important.

Photoresistors (a) are generally semiconducting substances in which extra carriers are created when illuminated with light whose photon energy exceeds its band gap. The extra carriers lower the electrical resistance of the substance. Although photoresistors are cheap, their change in resistance is not directly proportional to light intensity.

Photodiodes (b) are devices consisting of a junction of two semiconducting substances, most commonly *p* and *n* type silicon. When light is absorbed in the *depletion layer* between the *p* and *n* type silicon, a hole and an electron carrier are created and are drawn apart by the intense electric field in the depletion layer. Silicon photodiodes can measure light with wavelengths in the range 400 nm to 1050 nm. Although not equally sensitive to all wavelengths in this range, for any particular wavelength, the current is directly proportional to the light intensity. This is also the basic process used in silicon solar cells used to generate electricity from sunlight. See also Figure 7.56.

Photomultipliers (c) consist of a series of specially-coated electrodes inside a vacuum container. As explained in the text, a photon of light first liberates a few electrons at the first cathode. These electrons are then accelerated towards a second electrode where they release secondary electrons. These are then accelerated and after 7 or 8 electrodes, a detectable current pulse is produced in response to even a single photon.



Photomultipliers

The most sensitive of all light-detecting devices are *photomultipliers*. These are evacuated glass tubes with a series of metal plates, *cathodes*, held at different voltages. Light illuminates the first plate, called the *photocathode*. By means of the photoelectric effect, a photon incident upon the photocathode can cause the emission of an electron. The electric field around the *photocathode* is such as to accelerate the emitted electron towards the second cathode. As the electron strikes the second cathode, it causes the emission of typically 6 or 7 *secondary electrons*, which are then drawn towards the third cathode. Here each of these secondary electrons causes a further 6 or 7 electrons to be emitted. Thus between $6^2 = 36$ and $7^2 = 49$ electrons are drawn towards a fourth cathode, and so on. A photomultiplier might contain as many as ten cathodes, after which the number of electrons has increased to between $6^{10} \approx 6.0 \times 10^7$ and $7^{10} \approx 2.8 \times 10^8$.

Thus, each photon produces a current pulse of typically 10^8 electrons. Although this amplification is impressive, it still presents a measurement challenge. In terms of voltages, it represents a voltage pulse of perhaps 1 mV lasting for a fraction of a microsecond, on top of the total voltage across the photomultiplier tube of around 1000V.

Interferometry

Measurement of distances or of motion can be achieved by using the techniques of optical interferometry with a resolution of less than a wavelength of light $\approx 7 \times 10^{-7}$ m, almost independent of the distance in question. The use of these tech-

niques is made enormously more straightforward by the existence of cheap and robust sources of coherent light: lasers.

The basic principle of interferometry is illustrated in Figure 3.7. Light from a source S travels along two (or more) paths, say A and B , to a detector D .

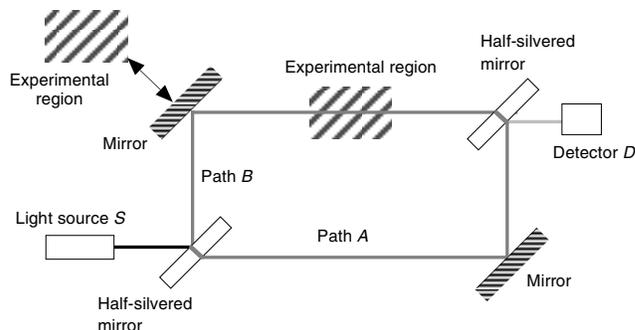
First of all recall that light is the name we give to oscillations of the electric field when the oscillations occur with frequencies in the range 400×10^{12} Hz (blue) and 1000×10^{12} Hz (red). The electric field at D is the sum of the electric fields due to light arriving via path A and path B . If light from a source S follows a route which passes a point D via point A then the electric field at D oscillates rapidly with amplitude E_A volts per metre. If light from S travels to D by a second route, say via B , then the electric field at D is the sum of two oscillating electric fields, one with amplitude E_A and the other with amplitude E_B .

If the light from S is derived from several random sources of light within S (such as the atoms within a light bulb) then the electric field at D due to the waves arriving via A and B will sometimes add up, and sometimes subtract. On average this results in a value of electric field amplitude greater than the either E_A or E_B independently, but not equal to the sum $E_A + E_B$. Importantly, the average resultant amplitude of the electric field oscillation, given by:

$$\sqrt{[E_A^2 + E_B^2]}$$

does not depend on the precise lengths of paths A and B , or on the *details* of the conditions experienced along those paths.

Figure 3.7 Illustration of the use of an optical interferometer. Experiments placed in either of the experimental regions can affect the interference of light from source S at detector D . To detect small movements an experiment could be arranged to move a mirror in the optical path. To detect changes in optical properties of a material, the material could be placed directly in the optical path



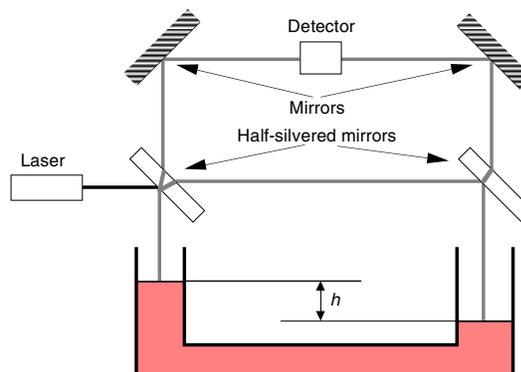
However, things are different if the light from S is derived from a *coherent* source of light, i.e. one in which all the atoms in S emit light *in phase* with each other. In this case, the waves arriving at D will add up *on every oscillation*, subtract *on every oscillation*, or do something in between *on every oscillation*. Exactly which situation occurs depends on the details of the conditions experienced along the paths A or B (for example, the presence or absence of a gas) and on the precise length of the paths. For a given set of conditions along the paths, the amplitude of the electric field oscillations varies between 0 and $(E_A + E_B)$. Let us say, for example, that under a given set of conditions along paths A and B the electric field oscillations have zero amplitude: then either:

- a shift in the position of the mirror by approximately one half a wavelength of light ($\approx 10^{-7}$ m) or
- a delay of the light wave by $\approx 10^{-15}$ s caused by a change in some property of the medium through which the light travels,

is sufficient to change the amplitude of the electric field oscillations at D from zero amplitude to maximum amplitude $(E_A + E_B)$.

There are many examples of the use of laser interferometry but space permits the inclusion of only two. The first example (Figure 3.8) is the detection of changes in the height of a liquid surface, and the second (Figure 3.9) is the direct measurement of the optical properties of a gas.

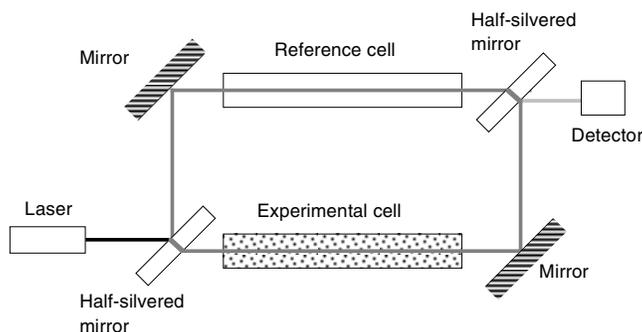
Figure 3.8 An optical interferometer used for the determination of change in height of a column of mercury.



In measurements of *pressure* it is common to use a U-tube filled with liquid as a simple manometer. This translates pressure differences into changes in the heights of liquid in the arms of the manometer. Using an optical interferometer the change in the liquid height can be detected with a resolution of the order 10^{-7} m.

The speed of light travelling through a gas differs slightly from the speed of light in a vacuum. As is discussed more fully in §5.8.2, the speed difference is related to the polarisability of the molecules of the gas. The device shown in Figure 3.9 is designed to measure small changes in the speed of light as the pressure of the gas in an experimental cell is changed (§5.8).

Figure 3.9 An optical interferometer used for the determination of the electrical polarisability of a gas. The apparatus is first set to a reference state by evacuating both the reference cell and the experimental cell. The reference cell is then kept under vacuum while the gas under investigation is introduced into the experimental cell. The light travelling through the experimental cell is then slowed down slightly by the interaction of its oscillating electric field with the electric charge on each atom. This results in a change in the intensity of light at the detector because the interference condition now depends on the transit time through the experimental cell.



3.3.4 Bridges and balances

In studying the properties of matter, it is an unfortunate fact that many substances show only relatively small changes in their properties in response to changes in their environment. Consider the examples *A* and *B* below.

Example A

The volume of solids and liquids changes by typically 1 part in 10^5 when the temperature changes by 1°C . The thermal expansion of most solids is of a similar order of magnitude, and so in order to compare the thermal expansion of different materials, we need to detect a length change of 1 part in 10^5 with a resolution of, say, 1% or better. In other words, we need an apparatus that will detect a length change of 1 part in 10^7 . In an experimental sample that might be $\approx 1\text{cm} \times 1\text{cm} \times 1\text{cm}$, a change of 1 part in 10^7 corresponds to detecting a length change of $\approx 10^{-9}$ m, or about 3 atomic diameters.

Example B

In order to determine the heat capacity of a substance we need to change the temperature by a small amount ΔT at an absolute temperature of T . Ideally ΔT will be a small fraction T , typically perhaps 1%. But ΔT must be measured with a resolution of ≈ 1 part in 10^3 in order to determine the heat capacity with an accuracy of 0.1%. Thus it seems that the temperature must be measured to 1 part in 10^5 .

Situations such as these, and a variety of others, call for the application of *comparison* or *bridge techniques*. What these techniques have in common is that they are sensitive not to the static background quantity (e.g. length or temperature in our examples), but to *small changes* in it.

Wheatstone bridge

The simplest example of this is the case of the *Wheatstone Bridge* electrical circuit shown in Figure 3.10.

The circuit, consisting of four resistors, a voltmeter and a voltage source, has two key features of its response.

- The circuit is said to be *balanced* when $R_a = R_b$ and $R_c = R_d$. At this point there will be no potential difference across the detector **V**. Thus the voltmeter is not sensitive to the magnitude of any of the resistances in the circuit, but it is sensitive to whether the resistances are *balanced*.
- Suppose that the circuit is initially balanced, and then R_d changes slightly from its balance condition by an amount ΔR . The voltmeter may be used on its most sensitive range to detect the out-of-balance voltage, which is given by:

$$\Delta V = i_{\text{bridge}} \left[\frac{\Delta R (R_a + R_b)}{R_a + R_b + R_c + R_d} \right] \quad (3.4)$$

where we have assumed $\Delta R \ll R_a + R_b + R_c + R_d$. Suppose the voltmeter can resolve a change of

Figure 3.10 The Wheatstone Bridge circuit. A voltage source drives a current i_{bridge} through the resistance network illustrated. The detector **V** need not have, but most commonly will have, an impedance much greater than any of the other resistances in the circuit. The bridge is balanced when $R_a = R_b$ and $R_c = R_d$.

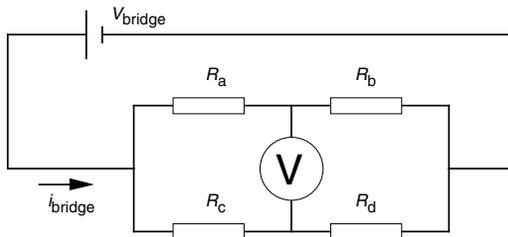
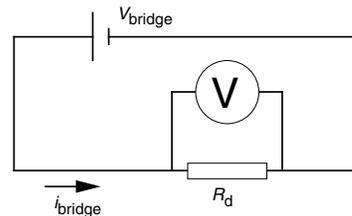


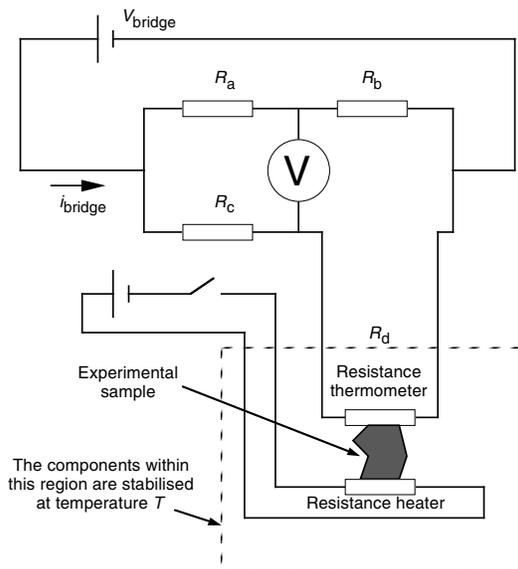
Figure 3.11. The alternative to a Wheatstone Bridge circuit. A voltage source drives a current i_{bridge} through the resistance R_d . The detector **V** must have an impedance much greater than R_d . If R_d changes by ΔR then the voltage across R_d changes by $\Delta V = i_{\text{bridge}} \Delta R$.



1 μV in 100 mV, a common specification. If R_a , R_b , R_c , and R_d were all $1000\ \Omega$, $\Delta R_d = 1\ \Omega$ and $i_{\text{bridge}} = 10\ \text{mA}$, then ΔV would be 5 mV. The voltmeter would then be able to detect a change of $1\ \Omega$ in a total resistance of $1000\ \Omega$ with a resolution of 1 part in 5000. The bridge circuit would thus just be able to detect resistance changes of the order of one thousandth of an ohm, i.e. 1 part in 10^6 of the total resistance.

Notice that the voltage change at the detector in a bridge circuit (5 mV in the above example) is less than the voltage change which would occur if we passed the entire current i_{bridge} through the resistor R_d (Figure 3.11). If we did this we would get a change in voltage $\Delta V = i_{\text{bridge}} \Delta R$. This would be equal to 10 mV in the above example. So why bother with the complication of a bridge circuit? When using the bridge, the signal is 5 mV, increased from nothing, i.e. $0\ \text{mV} \Rightarrow 5\ \text{mV}$. However, in the direct measurement configuration, the 10 mV is on top of a 1 V background, i.e. $1.000\ \text{V} \Rightarrow 1.010\ \text{V}$. Although the bridge has halved the voltage due to the resistance change, the 5 mV signal is easier to measure with high resolution

Figure 3.12 The Wheatstone bridge may be used to detect small changes in a resistance thermometer due to a temperature change ΔT .



because the background signal has been subtracted.

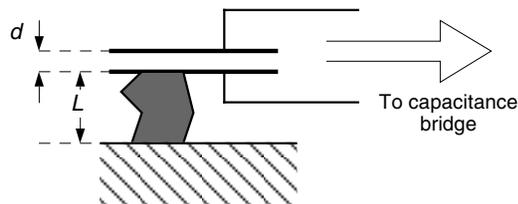
This is exactly the technique used for example *B* above. The element R_d of the circuit is arranged to be a resistance thermometer. The temperature step ΔT is applied by a resistive heater and results in a small change ΔR in the value of the resistance thermometer. The magnitude of the change ΔR is inferred from measurement of ΔV , and then converted to an equivalent temperature change ΔT .

Similarly, for example *B* above changes in the sample length can be converted into changes in the separation of two metal plates forming a capacitor (Figure 3.13). The changes in capacitance can be detected with high resolution by a circuit called a *capacitance bridge*, which is analogous to the Wheatstone bridge but which operates with alternating current.

The examples above represent ways of detecting *small changes* in quantities in a way that is insensitive to the magnitude of the quantities themselves.

The use of bridges has one more advantage. Consider again Equation 3.4 and the circuit it refers to in Figure 3.10. Suppose that instead of measuring the out-of-balance voltage ΔV , we adjusted R_c so as to restore the balance condition $\Delta V = 0$. This would allow us to determine ΔR in terms of the four resistances R_a, R_b, R_c, R_d and the only requirement placed on the detector is that it should show a true zero. It does not need to be linear or even calibrated! There are occasions where this relatively simple approach produces results with the lowest measurement uncertainty.

Figure 3.13 A schematic view of a capacitance cell. Small changes in the length of a sample cause changes in the separation between the two plates of a capacitor.



3.4 Environments

As well as being able to measure different physical quantities, we also need to be able to create and maintain environments in which we can perform experiments on samples of matter.

3.4.1 Temperature

High temperatures

Temperatures above room temperature are usually created by ovens or furnaces that are heated electrically. It is not difficult or particularly expensive to create an environment of a volume of 10 cm^3 or so with temperatures up to $900\text{ }^\circ\text{C}$. For temperatures above this, special materials (e.g. platinum or silicon carbide) must be used to resist the high rates of oxidation which would otherwise reduce the material of the heating filament to a powder. An alternative way to avoid oxidation is to operate a furnace either in vacuum or with an inert atmosphere. For example, graphite is commonly used as both as a heater and as a constructional material in furnaces operating up to $2000\text{ }^\circ\text{C}$ in vacuum. However, it would burn rapidly if any oxygen were present.

Low temperatures

Low temperature environments are usually created by placing experiment samples in contact with a cold fluid. For temperatures only slightly below room temperature, we can use liquid refrigerants that are similar to those found in a domestic refrigerator. For lower temperatures it is common to use liquid nitrogen, which boils at atmospheric pressure at a temperature of around 77 K ($-197\text{ }^\circ\text{C}$). For still lower temperatures, the refrigerant used is liquid helium, which boils at atmospheric pressure at a temperature of around 4.2 K ($-269\text{ }^\circ\text{C}$). By lowering the vapour pressure above the liquid, we can reduce the boiling temperature to $\approx 1.2\text{ K}$, which suffices for most purposes.

3.4.2 Pressure

The pressure under which an experiment is performed may be altered from below atmospheric pressure $\approx 10^5\text{ Pa}$, to values as large as 10^{10} Pa (10 GPa). The devices used to change the pressure are all based on, or adapted from, the *piston* (Figure 3.14).

Figure 3.14 Example of the use of piston to create pressure environments. In (a) a force F acts over area A to produce a pressure $P_1 = F/A$. In (b) the force acts over a smaller area $A/4$ to produce a pressure $4P_1$, even though the force used is the same in both cases.

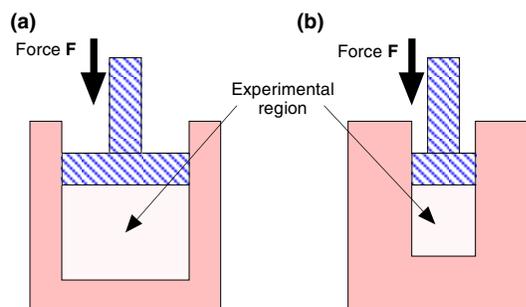


Figure 3.15 Illustration of techniques for applying high pressures to solid samples. The technique (a) uses a fluid around the sample to ensure that the pressure is transmitted hydrostatically. For application of the very highest pressures (b) the hardest material known, diamond, is used as a pressure anvil. The transparency of diamond allows relatively easy optical access. However the expense of large unflawed diamonds limits the size of the apparatus illustrated in (b), and all such experiments represent a considerable challenge to the experimenter.

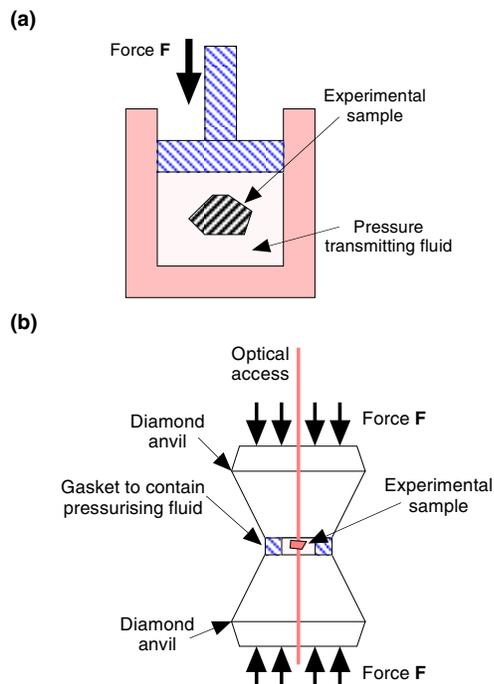
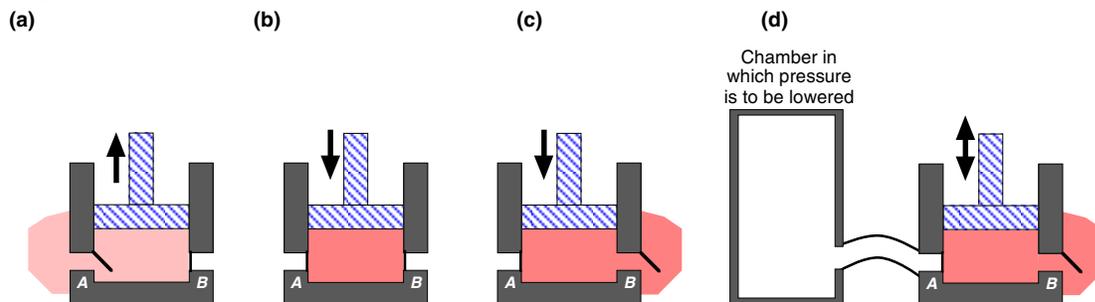


Figure 3.16 Illustration of the principle of operation of a mechanical vacuum pump. In (a) a piston/chamber is enlarged and 'sucks in' gas through *A* from the region in which the pressure is to be lowered. In (b) the connection *A* to the area in which the pressure is to be lowered is closed, and the piston lowered to compress the gas until the pressure exceeds atmospheric pressure (c), when a valve *B* opens to permit the compressed gas to be expelled. This process (d) can be used to lower the pressure in an experimental enclosure. Note: Modern vacuum pumps usually operate on a rotational rather than a reciprocating cycle as shown in (a) to (d) and they rarely look anything like this diagram might intimate.



For gases and liquids it is a straightforward extension of Figure 3.14 to see how pistons can be used to apply pressure. The experimental sample is placed in a suitable container, and a force applied to the piston. The force is often amplified by means of a lever or a hydraulic arrangement. At high pressures it is often a matter of considerable ingenuity to extract measurements from within the high pressure environment, but that need not concern us here.

For solids, it is important to ensure that pressure is applied uniformly in all directions, otherwise samples are easily damaged. To ensure that pressure is *hydrostatic* a pressure-transmitting fluid which does not itself solidify under pressure is used (Figure 3.15).

Achieving low pressures is no less a matter for ingenuity. Figure 3.16 shows schematically how the pressure may be lowered within a chamber by a simple mechanical *vacuum pump*, generally called a *rotary* or *roughing* pump. Devices operating on this principle can fairly easily reduce the pressure in a chamber to around 10 Pa. This is one ten thousandth part of atmospheric pressure, and corresponds to the removal of 99.99% of all the molecules in the chamber. However there would still typically be 10^{21} molecules left per cubic metre (see Example 4.2). A number of devices are available which can achieve so-called high vacuum of 10^{-4} Pa, and with great care and expense, *ultra high vacuum* pressures of around 10^{-10} Pa may be achieved.

3.5 Uncertainty

In most of the tables in the following chapters, there is no mention of uncertainty. Of course, all measurements are accompanied by uncertainty, and strictly speaking the uncertainty of all the measurements in the book should be stated. The reason that statements of measurement uncertainty are lacking is that most of the sources from which I have obtained the data do not quote uncertainties. As a guide, all the figures I give in measured quantities are usually significant figures. For ex-

ample, the density of the element Hafnium is given in Table 7.1 as 13276 kg m^{-3} . A realistic uncertainty for this is likely to ± 2 or 3 in the last figure given.

The bibliography in Chapter 1 contains the sources of the data used in the tables and figures in this book. You should consult these sources, and go beyond them to the original research papers if you wish to establish the uncertainties of the measurements with greater... certainty!

3.6 Exercises

Exercises marked with a P prefix are ‘normal’ exercises. Those marked with a C prefix are best solved numerically by using a computer program or spreadsheet. Exercises marked with an E prefix are in general rather more challenging than the P and C exercises. Answers to all the exercises may be downloaded from www.physicsofmatter.com

Some more demanding exercises relating to material in this Chapter and others may be found in Chapter 12.

Units

P1. What are the SI units (Tables 3.1 and 3.3) for the specification of:

- Magnetic flux density
- Temperature
- Electrical conductance
- Electrical resistance
- Electrical capacitance
- Amount of substance
- Mass

P2. What is the mass of (a) 1 mole of carbon atoms, (b) 1 mole of nitrogen atoms, and (c) 1 mole of nitrogen molecules (N_2).

P3. Given that the density of platinum/iridium is approximately $22 \times 10^3 \text{ kg m}^{-3}$. Roughly how big is the kilogram \mathfrak{K} ? Assuming that \mathfrak{K} is a cube (which it is not) how thick is a 1 microgram layer of H_2O ? You should find a result which is surprisingly thin. How many layers of H_2O molecules would be needed to make such a layer?

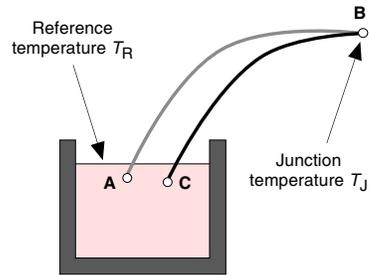
Measurement techniques

P4. If the frequency standard of NPL (§3.3.1) were used as the basis of a clock, how many years would it take before the time as determined by the clock was uncertain by 1 second? If the clock lasted for 100 years before it had to be replaced, how uncertain would the time be when the clock was replaced?

P5. I have read in *Scientific American* magazine that the United States of America and the Europe are moving apart from one another at a rate around one centimetre per year. How many metres per second does this correspond to? How many extra rows of atoms per second appear between the USA and Europe? How does this compare with the speed at which your hair grows? Suggest how, given a sufficient research

budget, you would attempt to confirm this result. How long would your measurement take?

P6. We mentioned in §3.3 that the voltage generated by a thermocouple did not arise at the junction between the two dissimilar metals. Instead it arose all along the wires and depended on the product of the so-called *Seebeck coefficient* $S(x)$ for a material and the temperature *gradient* dT/dx at each point on the wire.



In the figure above, points A and C are at the same reference temperature T_R and point B is the measurement junction with temperature T_J . A is connected to B by wire with Seebeck coefficient S_1 and C is connected to B by wire with Seebeck coefficient S_2 . The voltage V_{AC} is given by:

$$V_{AC} = \int_A^C S(x) \frac{dT}{dx} dx$$

Show that if the Seebeck coefficients are constant along each type of wire, that V_{AC} is indeed proportional to the temperature difference between T_R and T_J and given by:

$$V_{AC} = [T_R - T_J] \times [S_2 - S_1]$$

P7. Suggest a practical measurement application for (a) a slow-reacting photo-resistor costing 10 pence and (b) a fast-reacting (within a microsecond) photodiode costing £1.

P8. In the experiment on which I am presently working at the NPL, I have to measure quite precisely the amount of light which is Rayleigh-scattered by a gas. Rough 0.1 nW of green light with a wavelength of 532 nm falls onto my detector. The detector (known as a *trap detector*) is a combination of three silicon photodiodes carefully arranged to capture essentially all incoming radiation. Assuming that one photoelectron is generated for each photon that falls on the detector, how much current should I expect to flow from my detector. If I had used a photomultiplier instead of a photodiode how much current might I reasonably have expected? So why have I used a photodiode?!

P9. On a bright moonlit night it is quite possible to see one's way home from a friend's house. How many optical photons per second are striking each square metre of the Earth in order to make this possible? I suggest you assume that 1 kW m^2 of entirely optical energy (average frequency $7 \times 10^{14} \text{ Hz}$) strikes both the Earth and the Moon from the Sun. Assume that the light striking the Moon is reflected with 100% efficiency. The Moon–Earth separation is approximately $384 \times 10^6 \text{ m}$ and Sun–Earth (or Sun–Moon) separation is $150 \times 10^9 \text{ m}$. The Moon's diameter is approximately $3.5 \times 10^6 \text{ m}$.

P10. There is a link which I have not brought out in this book between the measurement of temperature (§3.3.2) and the detection of optical radiation (§3.3.3). This arises because all surfaces emit radiation which is characteristic of (a) their temperature and (b) an intrinsic surface property known as the *emissivity*. If the emissivity of a surface is known (which in practice it rarely is!), then its temperature can be inferred simply by exposing a detector to radiation coming from the surface. Such devices are the basis of many *non-contact thermometers*. The emission into a hemisphere around a surface of area A of a body with emissivity ε is given by $P = A\varepsilon\sigma T^4$ where σ is the *Stephan–Boltzmann* constant with the value $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$. The Sun has a radius of $7 \times 10^8 \text{ m}$ and the Sun–Earth separation is $150 \times 10^9 \text{ m}$. roughly 1 kW m^{-2} of radiant energy arrives at the Earth from the Sun. Estimate the temperature of the surface of the sun assuming it emits like a so-called *black body* with emissivity $\varepsilon = 1$

Assuming your body has a surface temperature of 22°C , how much energy is radiated from your skin every second?

P11. An apparatus similar to Figure 3.9 is constructed. Show that the number of wavelengths of light in the reference cell of length L may be expressed as $Ln_{\text{light}}/\lambda_0$ where n_{light} is the refractive index of the gas and λ_0 the wavelength of the light in free space. Using:

- $n = P/k_B T$ from Example 4.2 where n is the number density of molecules
- Equation 2.17
- Equation 5.109 and Table 5.17

show that the refractive index of a gas is given approximately by $1 + P\alpha/2\varepsilon_0 k_B T$ where P and T are the temperature and pressure of the gas. Hence, describe how α , the molecular polarisability of an individual gas molecule, may be determined. If you think about it I hope you will find even the possibility of this kind of measurement very impressive.

P12. Verify Equation 3.4 for a Wheatstone bridge near to the balance condition. What assumption has been made about the internal resistance of the voltmeter? Compare the magnitude of the voltage change ΔV with the voltage change $i_d \Delta R_d$ across R_d and state what advantages a bridge circuit offers.

P13. A capacitance bridge (Figure 3.13) can detect a change of capacitance of 1 part in 10^8 . If used in conjunction with a parallel-plate capacitor ($C = \varepsilon_0 A/d$) of area $A = 1 \text{ cm}^2$ and separation $d = 0.1 \text{ mm}$, estimate the smallest length change that can be detected. Roughly what fraction of the diameter of an atom does that correspond to? Could this sensitivity really be achieved in practice?

P14. A mass of 10 kg is dropped through 1 metre onto the blunt end of a tapered piece of steel. The blunt end of the steel has a diameter of 30 mm and the sharp end of the steel has a diameter of 0.2 mm . Estimate the maximum pressure under the tip of the taper. Could such a device be constructed on similar principles for achieving high pressures in a laboratory environment?

P15. A mechanical vacuum pump such as that described in §3.4 removes a fixed *volume* of gas from a chamber each second. This volume (known technically as the *speed* S of the pump) is just the volume of the chamber times the number of cycles per second that the pump piston executes. As the pressure of the gas in the chamber is lowered, a fixed volume of gas contains fewer and fewer molecules at the rate of reduction of pressure slows down. It can be shown that the pressure P in the chamber is described by:

$$P = P_0 \exp\left(-\frac{St}{V}\right)$$

where P_0 is the initial pressure, V is the volume of the chamber being evacuated, S is the pump speed and t is the time.

A modest mechanical vacuum pump might have a speed of $8 \text{ m}^3 \text{ hour}^{-1}$. Estimate the time to lower the pressure in a chamber of volume 100 litres from 1 atmosphere (10^5 Pa) to 1 Pa . This is about the low pressure limit for such a pump. At this pressure, the vapour pressure of the oil used to cool the moving piston is equal to the pressure in the chamber and net flow of molecules from the chamber is zero.

Although it requires further research outside the scope of this book, you might be interested to find out how such a chamber be evacuated to a pressure of 10^{-3} Pa or lower? The lowest pressure achievable is around 10^{-8} Pa which takes exceptional effort to achieve.

